



## Basic Concept

**Integer:** complete numbers are called integers

Such as -3, -2, -1, 0, 1, 2 .....etc.

They are represented by **I** or **Z**.

**Natural Numbers:** positive integer are called natural numbers they are represented by

$N = 1, 2, 3, 4 \dots\dots$

**Whole Numbers:** non-negative integers are called whole numbers they are represented by

$W = 0, 1, 2, 3, \dots\dots$

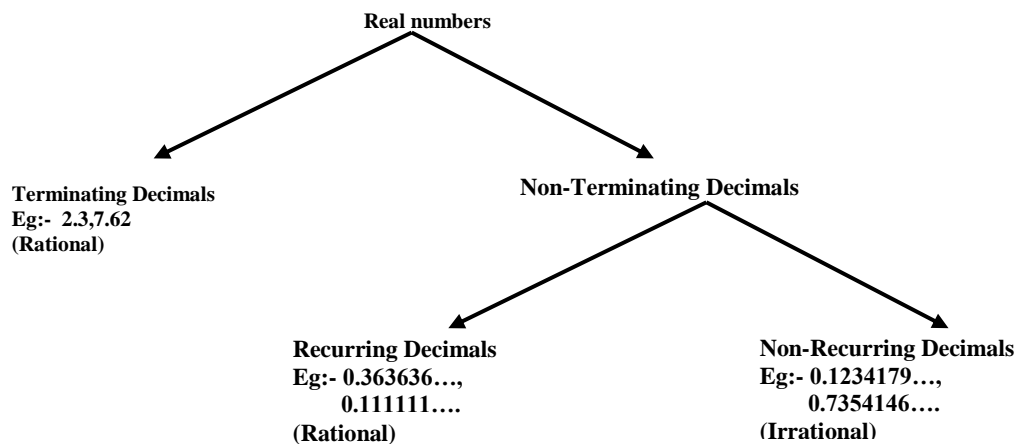
**Rational Numbers:** Numbers those can be written in the form of  $p/q$  where p, q are integers

Such as  $2/3, 1/3, 1/2 \dots$ etc. They are represented by **Q**

**Irrational numbers:** Numbers those are not rational are called irrational numbers such as  $\sqrt{3}, \sqrt{2}, \sqrt{5}, \pi, \dots$

**Real Numbers:** all rational and irrational numbers are called real numbers they are represented by **R**.

$$\text{Real} = \text{Rational} + \text{Irrational}$$



**Note:** (i) Integers are discrete numbers and real numbers are continuous numbers.

(ii) Discrete numbers are written in the form of the set and continuous numbers are written in the form of intervals.

**Prime numbers:-** positive integers which are divisible by only 1 or itself are called prime numbers, eg: 2,3,5,.....

Ex. If p is a positive integer then prove that both (n+3) and (n+8) cannot be simultaneously prime numbers.

**Composit numbers:-** Integers which are divisible by any other number other than 1 and itself are composite numbers such as 4,9,12,.....

**Coprime Numbers:-** Two positive integers a and b are co-prime if and only if HCF of a,b is 1, eg: 7 and 9.

## Inequality

These signs are use for comparison of two real numbers. Complex numbers can not be compared except equality. There are four sign of inequality –

Let a and b are two real numbers

- |   |                |
|---|----------------|
| (1) $a > b$ “a is greater than b”             | Ex. $3 > 2$    |
| (2) $a \geq b$ “a is greater or equal to b”   | Ex. $3 \geq 2$ |
| (3) $a < b$ “a is less than b”                | Ex. $2 < 3$    |
| (4) $a \leq b$ “a is less than or equal to b” | Ex. $2 \leq 3$ |

### Properties of inequality:

- (1) Any real numbers can be added or subtracted to both side of an inequality

$$a > b$$

$$a \pm k > b \pm k$$

- (2) If a positive real number is multiplied both sides of an inequality then the inequality does not change

$$a > b \Rightarrow ak > bk ; k \in R^+$$

- (3) if a negative real number is multiplied both sides of an inequality then the inequality change

$$\Rightarrow a > b \Rightarrow -ak < -bk$$

(4) if both sides of an inequality are positive then the inequality will change by taking the reciprocal  $a > b \ a, b > 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$

(5) If both sides of an inequality are +ve then square root can be taken  $\left\{ \begin{matrix} a > b \\ \sqrt{a} > \sqrt{b} \end{matrix} \right\} \ a, b > 0.$

(6) If both sides of an inequality are +ve then square can be taken both sides  $\left\{ \begin{matrix} a > b \\ a^2 > b^2 \end{matrix} \right\} \ a, b > 0.$

(7) Two same inequalities can be added  $\left\{ \begin{matrix} a > b \\ c > d \\ a + c > b + d \end{matrix} \right\}.$

(8) **Note:** subtraction is not always true it is only possible if all the side are +ve.

(9) Two same inequality can be multiplied if both sides are +ve

$$a > b$$

$$c > d \quad a, b, c, d > 0.$$

$$ac > bd$$

**Ex:-** (i) Find the values of  $x$  for which  $-5 \leq \frac{2-3x}{4} \leq 9.$

(ii) Find the solution set of the inequation  $\frac{1}{2} \left( \frac{3}{5}x + 4 \right) \geq \frac{1}{3}(x - 6).$

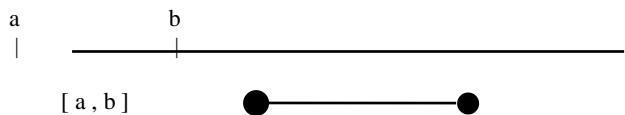
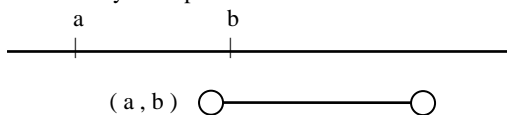
### Intervals :

(i) If  $a$  and  $b$  are two real no's such that  $a < b$ . Then set  $\{x: a < x < b\}$  is called the **open interval** from  $a$  to  $b$  and is written as  $(a, b)$ .

(ii) The set  $\{x: a \leq x \leq b\}$  is called

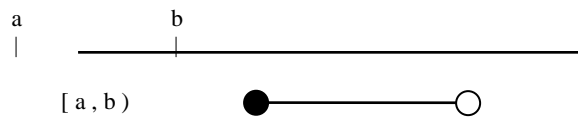
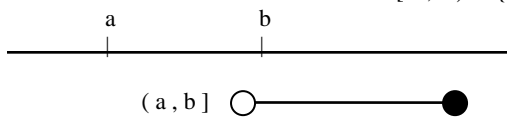
**closed interval** from  $a$  to  $b$  and is written as  $[a, b]$ .

On the no. line they are represented as



(iii) Similarly we define **semi open interval**  $(a, b] = \{x: a < x \leq b\}$

and **semi closed interval**  $[a, b) = \{x: a \leq x < b\}.$

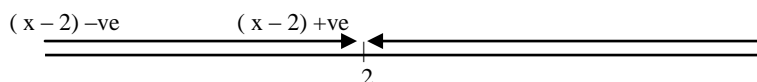


**Point of inversion :** let a factor  $(x - 2)$  where  $x$  is variables

$$(x - 2) < 0 \text{ -ve} \quad x > 2$$

$$(x - 2) > 0 \text{ +ve}$$

(i) if  $x < 2$



i.e. factor  $(x - 2)$  changes its sign in the neighborhood of '2' so 2 is the point of inversion of the factor  $(x - 2)$ .

**Absolute value :** the absolute value of any real no.  $a$  is denoted by  $|a|$  and it is defined as

$$|a| = \begin{cases} +a & a \geq 0 \\ -a & a < 0 \end{cases}$$

$$|2| = 2 \quad 2 \geq 0$$

$$|-2| = -(-2) = 2 \quad 2 < 0$$

**Note :** the absolute value of a real no. is always +ve.

**Logarithm:-**(exponential form)  $b^y = x \Leftrightarrow y = \log_b x$  (logarithmic form)Where  $b$  is called the base and  $x$  is argument. $y = \log_b x$  is defined if and only if  $x > 0$  and  $b > 0$  ( $b \neq 1$ )**Properties of logarithm:-**

1.  $\log_b 1 = 0$

2.  $\log_b b = 1$

3.  $\log_b (m \times n) = \log_b m + \log_b n$

4.  $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

5.  $\log_b m^n = n \log_b m$

6.  $\log_{b^n} m = \frac{1}{n} \log_b m$

7.  $\log_n m = \frac{\log_b m}{\log_b n}$  (base change formula)

8.  $\log_b a = \frac{1}{\log_a b}$

9.  $b^{\log_b m} = m$

10.  $\log_b a \cdot \log_c b \cdot \log_a c = 1$

**Greatest integer function:-**The function  $y = [x]$  is called the greatest integer function where  $[x]$  denotes the greatest integer just less than or equal to  $x$ , eg:

$[2.3] = 2, [-5.86] = -6, [5] = 5$

**Note:-**  $x - [x]$  is called the **fractional part of  $x$**  and is denoted by  $\{x\}$ , eg:  $\{2.3\} = 0.3, \{-5.6\} = 0.4, \{8\} = 0$ 

So any real number can be written as,

$$x = [x] + \{x\}$$

**Exercise****Inequality**

1. Solve the following inequality:-

(a)  $x^2 - 16 < 0$

(b)  $9 - x^2 > 0$

(c)  $x^2 - 5x + 6 > 0$

(d)  $x^2 - 10x + 21 < 0$

(e)  $x^2 + 2x - 15 \geq 0$

(f)  $2x^2 - 3x - 2 \leq 0$

(g)  $3x^2 - 10x + 3 \geq 0$

(h)  $x^2 - x + 1 < 0$

2. The number of integral solutions of  $x^2 - 3x - 4 < 0$ , is

(a) 3

(b) 4

(c) 6

(d) N.O.T.

3. If  $2 - 3x - 2x^2 \geq 0$ , then

(a)  $x \leq -2$

(b)  $-2 \leq x \leq \frac{1}{2}$

(c)  $x \geq -2$

(d)  $x \leq \frac{1}{2}$

4. The solution of  $6 + x - x^2 > 0$ , is

(a)  $-1 < x < 2$

(b)  $-2 < x < 3$

(c)  $-2 < x < -1$

(d) N.O.T.

5. Solve the following inequality:-

(a)  $\frac{x}{x+1} > 0$

(b)  $\frac{2x+1}{5-x} \geq 0$

(c)  $\frac{x-1}{x-2} > 2$

(d)  $\frac{x+2}{x-1} < 4$

(e)  $\frac{x^2 - 7x + 10}{x - 3} < 0$

(f)  $\frac{x^2 - 8x + 15}{x^2 - 9x + 14} < 0$

(g)  $\frac{x^2 - 4x - 21}{x^2 + 3x - 10} \leq 0$

(h)  $\frac{3x^2 + 7x + 4}{x^2 + x + 1} \geq 0$

6. The solution set of the inequation  $\frac{x+4}{x-3} \leq 2$ , is

(a)  $(-\infty, 3) \cup (10, \infty)$

(b)  $(3, 10]$

(c)  $(-\infty, 3) \cup [10, \infty)$

(d) N.O.T. .

7. The solution set of the inequation  $\frac{2x+4}{x-1} \geq 5$ , is

(a)  $(1, 3)$

(b)  $(1, 3]$

(c)  $(-\infty, 1) \cup [3, \infty)$

(d) N.O.T. .

8. The solution of the inequation  $\frac{4x+3}{2x-5} < 6$ , is

(a)  $(5/2, 33/8)$

(b)  $(-\infty, 5/2) \cup (33/8, \infty)$

(c)  $(5/2, \infty)$

(d)  $(33/8, \infty)$ .

9. The number of integral solutions of  $\frac{x+1}{x^2+2} > \frac{1}{4}$ , is

(a) 1

(b) 2

(c) 5

(d) N.O.T. .

10. The number of integral solutions of  $\frac{x+2}{x^2+1} > \frac{1}{2}$  is

(a) 4

(b) 5

(c) 3

(d) N.O.T. .

11. The solution set of the inequation  $\frac{x^2 - 3x + 4}{x + 1} > 1, x \in R$ , is

(a)  $(3, \infty)$

(b)  $(-1, 1) \cup (3, \infty)$

(c)  $[-1, 1] \cup [3, \infty)$

(d) N.O.T. .

12. The set of real values of  $x$  for which  $\frac{10x^2 + 17x - 34}{x^2 + 2x - 3} < 8$ , is

(a)  $(-5/2, 2)$

(b)  $(-3, -5/2) \cup (1, 2)$

(c)  $(-3, 1)$

(d) N.O.T. .

13. The set of real values of  $x$  for which  $\frac{8x^2 + 16x - 51}{(2x-3)(x+4)} < 3$ , is

(a)  $(3/2, 5/2)$

(b)  $(-4, -3)$

(c)  $(-4, -3) \cup (3/2, 5/2)$  (d) N.O.T. .

14. If  $S$  is the set of all real  $x$  such that  $\frac{2x}{2x^2 + 5x + 2} > \frac{1}{x+1}$ , then  $S$  is equal to

(a)  $(-2, -1)$

(b)  $(-2/3, 0)$

(c)  $(-2/3, -1/2)$

(d)  $(-2, -1) \cup (-2/3, -1/2)$ .

15. The solution set of  $x^2 + 2 \leq 3x \leq 2x^2 - 5$ , is

(a)  $\emptyset$

(b)  $[1, 2]$

(c)  $(-\infty, -1] \cup [5/2, \infty)$  (d) N.O.T. .

16. The greatest negative integer satisfying  $x^2 - 4x - 77 < 0$  and  $x^2 > 4$ , is

(a) -4

(b) -6

(c) -7

(d) N.O.T. .

17. If  $4 \leq x \leq 9$ , then

(a)  $(x-4)(x-9) \leq 0$

(b)  $(x-4)(x-9) \geq 0$

(c)  $(x-4)(x-9) < 0$

(d)  $(x-4)(x-9) > 0$ .

18. The set of all integral values of  $x$  for which  $5x - 1 < (x+1)^2 < 7x - 3$ , is

(a)  $\emptyset$

(b)  $\{1\}$

(c)  $\{2\}$

(d)  $\{3\}$  .

19. If  $x$  is an integer satisfying  $x^2 - 6x + 5 \leq 0$  and  $x^2 - 2x > 0$ , then the number of possible values of  $x$ , is

(a) 3

(b) 4

(c) 2

(d) infinite .

**Problem Based on Modulus**

Solve the following equations:-

(1)  $|x - 2| = 3$

(2)  $|x^2 - 4x| = 12$

(3)  $|2x + 5| = x$

(4)  $|x^2 - 6x + 3| = 1$

(5)  $x^2 - 3|x| + 2 = 0$

(6)  $x^2 + |x| - 6 = 0$

(7)  $x^2 - 5|x| + 6 = 0$

(8)  $|x^2 - 5x + 6| = x - 3$

(9)  $|x^2 + 2x - 8| + x - 2 = 0$

(10)  $|x^2 + 4x + 3| + 2x + 5 = 0$

(11)  $3^{|3x-4|} = 9^{2x-2}$

(12)  $|6x^2 - 5x + 1| = 5x^2 - 6x + 2$

**Logarithm**

1. What is the value of  $2 \log_8 2 - \frac{1}{3} \log_3 9$ ?

- (a)0 (b)1 (c)2 (d)1/3

2. What is the value of  $\log_y x^5 \log_x y^2 \log_z z^3$ ?

- (a)10 (b)20 (c)30 (d)60

3. If  $\log_{16} x + \log_4 x + \log_2 x = 14$ , then  $x$  is equal to

- (a)32 (b)64 (c)256 (d) N.O.T. .

4. What is  $\log(a + \sqrt{a^2 + 1}) + \log\left(\frac{1}{a + \sqrt{a^2 + 1}}\right)$  equal to

- (a)1 (b)0 (c)2 (d)
- $\frac{1}{2}$
- .

5.  $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$  is equal to

- (a)0 (b)1 (c)2 (d)N.O.T. .

6. If  $a^x = b, b^y = c, c^z = a$ , then the value of  $xyz$  is equal to

- (a)0 (b)1 (c)2 (d)3

7. If  $\frac{1}{\log_a x} + \frac{1}{\log_c x} = \frac{2}{\log_b x}$ , then  $a, b$  and  $c$  are in

- (a)AP (b)GP (c)HP (d)N.O.T. .

8. If  $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$ , then what is the value of  $y$ ?

- (a)4.5 (b)9 (c)18 (d)27

9. If  $x = \log_{2a} a, y = \log_{3a} 2a$  and  $z = \log_{4a} 3a$ , then  $xyz + 1 =$

- (a)
- $2yz$
- (b)
- $2xy$
- (c)
- $2zx$
- (d)N.O.T. .

10. If  $a, b, c$  are positive real numbers then  $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1} =$

- (a)0 (b)1 (c)2 (d)-1 .

11. If  $a = \log 2, b = \log 3, c = \log 7$  and  $6^x = 7^{x+4}$ , then  $x =$

- (a)
- $\frac{4b}{c+a-b}$
- (b)
- $\frac{4c}{a+b-c}$
- (c)
- $\frac{4b}{c-a-b}$
- (d)
- $\frac{4a}{a+b-c}$
- .

12. If  $a^2 + b^2 - c^2 = 0$ , then  $\frac{1}{\log_{c+a} b} + \frac{1}{\log_{c-a} b} =$

- (a)1 (b)2 (c)-1 (d)-2 .

13. If  $\log x = \frac{\log y}{2} = \frac{\log z}{5}$ , then  $x^4 y^3 z^{-2} =$

- (a)2 (b)10 (c)1 (d)0 .

14. If  $\frac{\log x}{2a+3b-5c} = \frac{\log y}{2b+3c-5a} = \frac{\log z}{2c+3a-5b}$  then  $xyz =$

- (a)2 (b)1 (c)0 (d)-1 .

15. What is the value of  $\log_{10} \left(\frac{9}{8}\right) - \log_{10} \left(\frac{27}{32}\right) + \log_{10} \left(\frac{3}{4}\right)$

- (a)3 (b)2 (c)1 (d)0 .

16. If  $\log_3 [\log_3 (\log_3 x)] = \log_3 3$ , then what is the value of  $x$

- (a)3 (b)27 (c)  $3^9$  (d)  $3^{27}$  .
17. What is the value of  $\frac{(\log_{27} 9)(\log_{16} 64)}{\log_4 \sqrt{2}}$
- (a)1 (b)2 (c)4 (d)8 .
18. If  $\log_k x \log_5 k = 3$ , then what is  $x$  equal to
- (a)  $k^5$  (b)  $5k^3$  (c)243 (d)125 .
19. If  $\log_{10} (x+1) + \log_{10} 5 = 3$ , then what is value of
- (a)199 (b)200 (c)299 (d)300 .
20. If  $\frac{\log_2 a}{2} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4}$  and  $a^{1/2} b^{1/3} c^{1/4} = 24$ , then
- (a)  $a = 24$  (b)  $b = 81$  (c)  $c = 64$  (d)  $c = 256$  .
21. If  $\frac{\log_a x}{\log_{ab} x} = 4 + k + \log_a b$ , then  $k =$
- (a)0 (b)1 (c)-2 (d)-3 .
22. If  $x = \log_a bc$ ,  $y = \log_b ca$ ,  $z = \log_c ab$ , then
- (a)  $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$  (b)  $\frac{1}{x-1} + \frac{1}{y-1} + \frac{1}{z-1} = 1$  (c)  $xyz = x + y + z + 1$  (d)  $xyz = 1$  .
23. If  $\log_4 5 = x$  and  $\log_5 6 = y$ , then  $\log_3 2$  is equal to
- (a)  $\frac{1}{2x+1}$  (b)  $\frac{1}{2y+1}$  (c)  $2xy + 1$  (d)  $\frac{1}{2xy-1}$  .
24. The value of  $x^{\log_a a \times \log_a y \times \log_a z}$  is
- (a)  $x$  (b)  $y$  (c)  $z$  (d)  $a$  .
25. If  $\log_{30} 3 = x$ ,  $\log_{30} 5 = y$ , then  $\log_{30} 8 =$
- (a)  $3(1-x-y)$  (b)  $x-y+1$  (c)  $1-x-y$  (d)  $2(x-y+1)$  .
26. If  $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$ , then  $x =$
- (a)4 (b)9 (c)83 (d)10 .
27. The value of  $a^{\frac{\log_b(\log_b x)}{\log_b a}}$ , is
- (a)  $\log_a x$  (b)  $\log_b x$  (c)  $\log_x a$  (d)  $\log_x b$  .
28. If  $a^x = b^y = c^z = d^w$ , then  $\log_a (bcd)$  equals to
- (a)  $\frac{1}{x} \left( \frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right)$  (b)  $x \left( \frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right)$  (c)  $\frac{y+z+w}{x}$  (d) N.O.T. .
29. If  $\log_5 (\log_5 (\log_2 x)) = 0$ , then the values of  $x$  is
- (a)32 (b)125 (c)625 (d)125 .
30. If  $y = 2^{1/\log_x 8}$ , then  $x$  equal to
- (a)  $y$  (b)  $y^2$  (c)  $y^3$  (d) N.O.T. .
31. If  $3^{2x+1} \cdot 4^{x-1} = 36$ , then  $x =$
- (a)  $\log_{36} 48$  (b)  $\log_{48} 36$  (c)  $\log_{24} 12$  (d)  $\log_{12} 24$  .
32. If  $\log_e 2 \cdot \log_x 27 = \log_{10} 8 \cdot \log_e 10$ , then  $x =$
- (a)1 (b)3 (c)2 (d)4 .
33. If  $3 + \log_5 x = 2 \log_{25} y$ , then  $x$  equals to

- (a)  $\frac{y}{125}$  (b)  $\frac{y}{25}$  (c)  $\frac{y^2}{625}$  (d)  $3 - \frac{y^2}{25}$
34. If  $5^{3x^2 \log_{10} 2} = 2^{\left(\frac{x+1}{2}\right) \log_{10} 25}$ , then  $x$  equals to  
 (a) 1. -3 (b) 1 (c)  $1, -\frac{1}{2}$  (d)  $-\frac{1}{3}, 1$
35. The value of  $\log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2$ , is  
 (a) 2 (b) 3 (c) 5 (d) 7
36. If  $a = 1 + \log_x yz, b = 1 + \log_y zx, c = 1 + \log_z xy$ , then  $ab + bc + ca =$   
 (a) 0 (b)  $2abc$  (c)  $abc$  (d)  $a^2 + b^2 + c^2$
37. If  $\log(2a - 3b) = \log a - \log b$ , then  $a =$   
 (a)  $\frac{3b^2}{2b-1}$  (b)  $\frac{3b}{2b-1}$  (c)  $\frac{b^2}{2b+1}$  (d)  $\frac{3b^2}{2b+1}$
38. If  $\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x}$ , then  $3^{x+y} 5^{y+z} 7^{z+x} =$   
 (a) 0 (b) 2 (c) 1 (d) N.O.T.
39. If  $\log(x+y) = \log 2 + \frac{1}{2} \log x + \frac{1}{2} \log y$ , then.  
 (a)  $x+y=0$  (b)  $x-y=0$  (c)  $xy=1$  (d)  $x^2 + xy + y^2 = 0$
40. If  $\log_a ab = x$ , then the value of  $\log_b ab$  is  
 (a)  $\frac{x-1}{x}$  (b)  $\frac{x}{x-1}$  (c)  $\frac{x}{x+1}$  (d)  $\frac{x+1}{x}$
41. If  $\log_{12} 27 = a$ , then  $\log_6 16$  is equal to  
 (a)  $2 \cdot \frac{3-a}{3+a}$  (b)  $3 \cdot \frac{3-a}{3+a}$  (c)  $4 \cdot \frac{3-a}{3+a}$  (d) N.O.T.
42. The value of  $(yz)^{\log y - \log z} \times (zx)^{\log z - \log x} (xy)^{\log x - \log y}$  is equal to  
 (a) 2 (b) 1 (c) 4 (d) 3
43. If  $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$ , then  $x^a \cdot y^b \cdot z^c$  is equal to  
 (a) 1 (b)  $abc$  (c)  $xyz$  (d) N.O.T.
44. If  $a = \log_{24} 12, b = \log_{36} 24, c = \log_{48} 36$ . Then  $1 + abc$  is equal to  
 (a)  $2ac$  (b)  $2bc$  (c)  $2ab$  (d) N.O.T.
45. If  $a, b, c$  are positive real numbers, then  $a^{\log b - \log c} \times b^{\log c - \log a} \times c^{\log a - \log b} =$   
 (a) 0 (b) 1 (c) -1 (d) N.O.T.
46. If  $a, b, c$  are three consecutive positive integers, then  $\log(1+ca) =$   
 (a)  $\log b$  (b)  $\log\left(\frac{b}{2}\right)$  (c)  $\log(2b)$  (d)  $2 \log b$
47. If  $\frac{1}{\log_2 a} + \frac{1}{\log_4 a} + \frac{1}{\log_8 a} + \frac{1}{\log_{16} a} + \dots + \frac{1}{\log_{2^n} a} = \frac{n(n+1)}{\lambda}$ , then  $\lambda$  equals  
 (a)  $\log_2 a$  (b)  $\log_a 4$  (c)  $\log_2 a^2$  (d) N.O.T.
48. The solution set of the equation  $\log_x 2 \cdot \log_{2x} 2 = \log_{4x} 2$ , is  
 (a)  $\{2^{-\sqrt{2}}, 2\sqrt{2}\}$  (b)  $\left\{\frac{1}{2}, 2\right\}$  (c)  $\left\{\frac{1}{4}, 4\right\}$  (d) N.O.T.

49. If  $\log_x (4x^{\log_5 x} + 5) = 2 \log_5 x$ , then  $x$  equals to

(a) 4,5

(b) -1,5

(c) 4,-1

(d)  $5, \frac{1}{5}$

**Greatest integer function**

**Solve the following equations:-**

1.  $[x] - 2 = 0$

2.  $[x]^2 - 5[x] + 6 = 0$

3.  $[x] + 5 = 0$

4.  $[x]^2 - 2[x] - 15 = 0$

5.  $[x]^2 = [x + 6]$

6.  $2[x] = [x + 3]$

7.  $[x]^2 - [x] - 20 = 0$

8.  $[x]^2 - 4[x] - 12 = 0$

9. If  $[x]$  denotes the greatest integer less than or equal to  $x$ , then the solutions of the equation  $2x - 2[x] = 1$  are

(a)  $x = n + \frac{1}{2}, n \in \mathbb{N}$

(b)  $x = n - \frac{1}{2}, n \in \mathbb{N}$

(c)  $x = n + \frac{1}{2}, n \in \mathbb{Z}$

(d)  $n < x < n + 1, n \in \mathbb{Z}$

10. If  $0 < x < 1000$  and  $[x]$  denotes the greatest integer less than or equal to  $x$ , then the number of possible values of  $x$  satisfying

$$\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] + \left[\frac{x}{5}\right] = \frac{31}{30}x,$$

(a) 34

(b) 32

(c) 33

(d) N.O.T.

11. If  $[x]^2 = [x + 2]$ , where  $[x]$  = the greatest integer less than or equal to  $x$ , then  $x$  must be such that

(a)  $x = 2, -1$

(b)  $x \in [2, 3)$

(c)  $x \in [-1, 0)$

(d)  $[-1, 0) \cup [2, 3)$

12. The value of  $[\sin x] + [1 + \sin x] + [2 + \sin x]$  in  $x \in (\pi, 3\pi/2]$  can be ([.] is the greatest integer function) can be

(a) 0

(b) 1

(c) 2

(d) 3

**Answer Sheet**

**Inequality**

1. (a)  $(-4, 4)$  (b)  $(-3, 3)$  (c)  $(-\infty, 2) \cup (3, \infty)$  (d)  $(3, 7)$  (e)  $(-\infty, -5] \cup [3, \infty)$  (f)  $\left[-\frac{1}{2}, 2\right]$

(g)  $\left(-\infty, \frac{1}{3}\right] \cup [3, \infty)$  (h)  $\emptyset$  2.b 3.b 4.b 5. (a)  $(-\infty, -1) \cup (0, \infty)$  (b)  $\left[-\frac{1}{2}, 5\right)$  (c)  $(2, 3)$

(d)  $(-\infty, 1) \cup (2, \infty)$  (e)  $(-\infty, 2) \cup (3, 5)$  (f)  $(2, 3) \cup (5, 7)$  (g)  $(-5, -3] \cup (2, 7]$  (h)  $\left(-\infty, -\frac{4}{3}\right) \cup [-1, \infty)$

6.c 7.b 8.a 9.c 10.c 11.b 12.b 13.c 14.d 15.a 16.b 17.a 18.d 19.a

**Modulus**

1. 5, -1 2. -2, 6 3. No solution 4.  $3 \pm \sqrt{7}, 3 \pm \sqrt{5}$  5.  $\pm 1, \pm 2$  6.  $\pm 2$

7.  $\pm 2, \pm 3$  8. 3 9.  $\{-5, -3, 2\}$  10.  $-4, -1 - \sqrt{3}$  11.  $\frac{8}{7}$  12.  $\frac{-1 \pm \sqrt{5}}{2}$

**Logarithm**

1.a 2.c 3.c 4.b 5.c 6.b 7.b 8.b 9.a 10.b 11.b 12.b 13.c 14.b  
 15.d 16.d 17.c 18.d 19.a 20.d 21.d 22.a 23.d 24.c 25.a 26.d 27.b 28.b  
 29.a 30.c 31.a 32.b 33.a 34.d 35.c 36.c 37.a 38.c 39.b 40.b 41.c 42.b  
 43.a 44.b 45.b 46.d 47.c 48.c 49.d

**Greatest integer function**

1.  $[2, 3)$  2.  $[2, 4)$  3.  $[-5, -4)$  4.  $[-3, -2) \cup [5, 6)$  5.  $[-2, -1) \cup [3, 4)$  6.  $[3, 4)$

7.  $[-4, -3) \cup [5, 6)$  8.  $[-2, -1) \cup [6, 7)$  9.c 10.c 11.d 12.a