(Physics)

0.1: Physical Constants

Speed of light	c	3×10^8 m/s
Planck constant	h	6.63×10^{-34} J s
	hc	1242 eV-nm
Gravitation constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k	1.38×10^{-23} J/K
Molar gas constant	R	8.314 J/(mol K)
Avogadro's number	N_A	6.023×10^{23} mol^{-1}
Charge of electron	e	1.602×10^{-19} C
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ N/A^2
Permitivity of vacuum	ϵ_0	8.85×10^{-12} F/m
Coulomb constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N m^2/C^2
Faraday constant	F	96485 C/mol
Mass of electron	m_e	9.1×10^{-31} kg
Mass of proton	m_p	1.6726×10^{-27} kg
Mass of neutron	m_n	1.6749×10^{-27} kg
Atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit	u	931.49 MeV/c^2
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/(m^2 K^4)
Rydberg constant	R_∞	1.097×10^7 m^{-1}
Bohr magneton	μ_B	9.27×10^{-24} J/T
Bohr radius	a_0	0.529×10^{-10} m
Standard atmosphere	atm	1.01325×10^5 Pa
Wien displacement constant	b	2.9×10^{-3} m K

1 MECHANICS

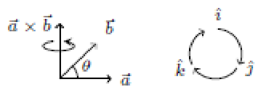
1.1: Vectors

Notation: $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$

Cross product:



$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

$$|\vec{a} \times \vec{b}| = ab\sin\theta$$

1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

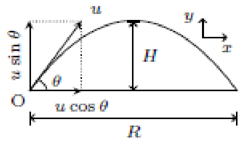
$$\vec{v}_{\text{av}} = \Delta\vec{r}/\Delta t, \qquad \vec{v}_{\text{inst}} = d\vec{r}/dt$$
$$\vec{a}_{\text{av}} = \Delta\vec{v}/\Delta t \qquad \vec{a}_{\text{inst}} = d\vec{v}/dt$$

Motion in a straight line with constant a:

$$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 - u^2 = 2as$$

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Projectile Motion:



$$x = ut\cos\theta, \quad y = ut\sin\theta - \tfrac{1}{2}gt^2$$
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

1.3: Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}, \quad \vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Frictional force: $f_{\text{static, max}} = \mu_s N, \quad f_{\text{kinetic}} = \mu_k N$

Banking angle: $\frac{v^2}{rg} = \tan\theta, \; \frac{v^2}{rg} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$

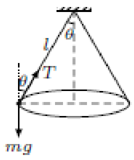
Centripetal force: $F_c = \frac{mv^2}{r}, \quad a_c = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{\text{pseudo}} = -m\vec{a}_0, \quad F_{\text{centrifugal}} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{\text{min, bottom}} = \sqrt{5gl}, \quad v_{\text{min, top}} = \sqrt{gl}$$

Conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$



1.4: Work, Power and Energy

Work: $W = \vec{F} \cdot \vec{S} = FS\cos\theta, \quad W = \int \vec{F} \cdot d\vec{S}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\text{gravitational}} = mgh, \quad U_{\text{spring}} = \tfrac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\text{conservative}} \cdot d\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

Mechanical energy: $E = U + K$. Conserved if forces are conservative in nature.

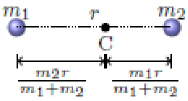
Power $P_{\text{av}} = \frac{\Delta W}{\Delta t}, \quad P_{\text{inst}} = \vec{F} \cdot \vec{v}$

1.5: Centre of Mass and Collision

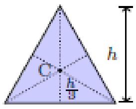
Centre of mass: $x_{\rm cm} = \frac{\sum x_i m_i}{\sum m_i}$, $\quad x_{\rm cm} = \frac{\int x\,dm}{\int dm}$

CM of few useful configurations:

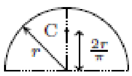
1. m_1, m_2 separated by r:



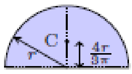
2. Triangle (CM $\equiv$ Centroid) $y_c = \frac{h}{3}$



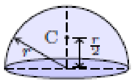
3. Semicircular ring: $y_c = \frac{2r}{\pi}$



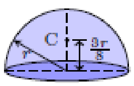
4. Semicircular disc: $y_c = \frac{4r}{3\pi}$



5. Hemispherical shell: $y_c = \frac{r}{2}$



6. Solid Hemisphere: $y_c = \frac{3r}{8}$



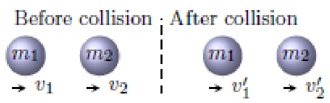
7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{\rm cm} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{\rm cm} = M\vec{v}_{\rm cm}, \quad \vec{a}_{\rm cm} = \frac{\vec{F}_{\rm ext}}{M}$$

Impulse: $\vec{J} = \int \vec{F}\,{\rm d}t = \Delta\vec{p}$

Collision:



Momentum conservation: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
Elastic Collision: $\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$
Coefficient of restitution:

$$e = \frac{-(v_1' - v_2')}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely in-elastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v_1' = -v_1$.
If $v_2 = 0$ and $m_1 \gg m_2$ then $v_2' = 2v_1$.
Elastic collision with $m_1 = m_2$: $v_1' = v_2$ and $v_2' = v_1$.

1.6: Rigid Body Dynamics

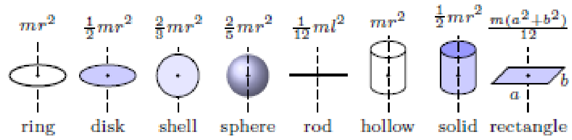
Angular velocity: $\omega_{\rm av} = \frac{\Delta\theta}{\Delta t}$, $\quad \omega = \frac{{\rm d}\theta}{{\rm d}t}$, $\quad \vec{v} = \vec{\omega} \times \vec{r}$

Angular Accel.: $\alpha_{\rm av} = \frac{\Delta\omega}{\Delta t}$, $\quad \alpha = \frac{{\rm d}\omega}{{\rm d}t}$, $\quad \vec{a} = \vec{\alpha} \times \vec{r}$

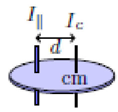
Rotation about an axis with constant α:

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega t + \tfrac{1}{2}\alpha t^2, \quad \omega^2 - {\omega_0}^2 = 2\alpha\theta$$

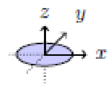
Moment of Inertia: $I = \sum_i m_i {r_i}^2$, $\quad I = \int r^2 {\rm d}m$



Theorem of Parallel Axes: $I_{\parallel} = I_{\rm cm} + md^2$



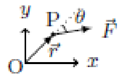
Theorem of Perp. Axes: $I_z = I_x + I_y$



Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$, $\quad \vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$, $\quad \vec{\tau} = \frac{{\rm d}\vec{L}}{{\rm d}t}$, $\quad \tau = I\alpha$



Conservation of $\vec{L}$: $\vec{\tau}_{\rm ext} = 0 \implies \vec{L} = $ const.

Equilibrium condition: $\sum \vec{F} = \vec{0}$, $\quad \sum \vec{\tau} = \vec{0}$

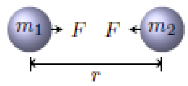
Kinetic Energy: $K_{\rm rot} = \frac{1}{2}I\omega^2$

Dynamics:

$$\vec{\tau}_{\rm cm} = I_{\rm cm}\vec{\alpha}, \quad \vec{F}_{\rm ext} = m\vec{a}_{\rm cm}, \quad \vec{p}_{\rm cm} = m\vec{v}_{\rm cm}$$
$$K = \tfrac{1}{2}m{v_{\rm cm}}^2 + \tfrac{1}{2}I_{\rm cm}\omega^2, \quad \vec{L} = I_{\rm cm}\vec{\omega} + \vec{r}_{\rm cm} \times m\vec{v}_{\rm cm}$$

1.7: Gravitation

Gravitational force: $F = G\frac{m_1 m_2}{r^2}$



Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

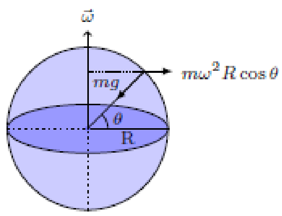
Variation of g with depth: $g_{\rm inside} \approx g\left(1 - \frac{2h}{R}\right)$

Variation of g with height: $g_{\rm outside} \approx g\left(1 - \frac{h}{R}\right)$

Effect of non-spherical earth shape on g:
$g_{\rm at\ pole} > g_{\rm at\ equator}$ ($\because R_e - R_p \approx 21$ km)

Effect of earth rotation on apparent weight:

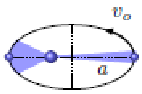
$$mg_\theta' = mg - m\omega^2 R\cos^2\theta$$



Orbital velocity of satellite: $v_o = \sqrt{\frac{GM}{R}}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Kepler's laws:



First: Elliptical orbit with sun at one of the focus.
Second: Areal velocity is constant. ($\because \mathrm{d}\vec{L}/\mathrm{d}t = 0$).
Third: $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM}a^3$.

1.8: Simple Harmonic Motion

Hooke's law: $F = -kx$ (for small elongation x.)

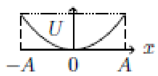
Acceleration: $a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{k}{m}x = -\omega^2 x$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

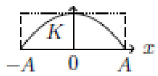
Displacement: $x = A\sin(\omega t + \phi)$

Velocity: $v = A\omega\cos(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Potential energy: $U = \frac{1}{2}kx^2$

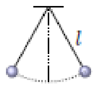


Kinetic energy $K = \frac{1}{2}mv^2$

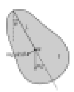


Total energy: $E = U + K = \frac{1}{2}m\omega^2A^2$

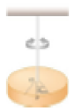
Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$



Physical Pendulum: $T = 2\pi\sqrt{\frac{I}{mgl}}$

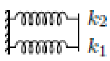


Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{k}}$

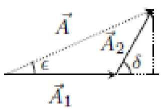


Springs in series: $\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$

Springs in parallel: $k_{\text{eq}} = k_1 + k_2$



Superposition of two SHM's:



$$x_1 = A_1\sin\omega t, \qquad x_2 = A_2\sin(\omega t + \delta)$$
$$x = x_1 + x_2 = A\sin(\omega t + \epsilon)$$
$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$
$$\tan\epsilon = \frac{A_2\sin\delta}{A_1 + A_2\cos\delta}$$

1.9: Properties of Matter

Modulus of rigidity: $Y = \frac{F/A}{\Delta l/l}$, $B = -V\frac{\Delta P}{\Delta V}$, $\eta = \frac{F}{A\theta}$

Compressibility: $K = \frac{1}{B} = -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}P}$

Poisson's ratio: $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta l/l}$

Elastic energy: $U = \frac{1}{2}$ stress $\times$ strain $\times$ volume

Surface tension: $S = F/l$

Surface energy: $U = SA$

Excess pressure in bubble:

$$\Delta p_{\text{air}} = 2S/R, \quad \Delta p_{\text{soap}} = 4S/R$$

Capillary rise: $h = \frac{2S\cos\theta}{r\rho g}$

Hydrostatic pressure: $p = \rho gh$

Buoyant force: $F_B = \rho Vg$ = Weight of displaced liquid

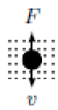
Equation of continuity: $A_1v_1 = A_2v_2$

Bernoulli's equation: $p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

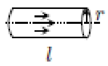
Torricelli's theorem: $v_{\text{efflux}} = \sqrt{2gh}$

Viscous force: $F = -\eta A\frac{\mathrm{d}v}{\mathrm{d}x}$

Stoke's law: $F = 6\pi\eta rv$



Poiseuilli's equation: $\frac{\text{Volume flow}}{\text{time}} = \frac{\pi pr^4}{8\eta l}$



Terminal velocity: $v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$