

## 4 Heat and Thermodynamics

### 4.1: Heat and Temperature

Temp. scales:  $F = 32 + \frac{9}{5}C$ ,  $K = C + 273.16$

Ideal gas equation:  $pV = nRT$ ,  $n$  : number of moles

van der Waals equation:  $(p + \frac{a}{V^2})(V - b) = nRT$

Thermal expansion:  $L = L_0(1 + \alpha\Delta T)$ ,  
 $A = A_0(1 + \beta\Delta T)$ ,  $V = V_0(1 + \gamma\Delta T)$ ,  $\gamma = 2\beta = 3\alpha$

Thermal stress of a material:  $\frac{F}{A} = Y \frac{\Delta l}{l}$

### 4.2: Kinetic Theory of Gases

General:  $M = mN_A$ ,  $k = R/N_A$

Maxwell distribution of speed:



RMS speed:  $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$

Average speed:  $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$

Most probable speed:  $v_p = \sqrt{\frac{2kT}{m}}$

Pressure:  $p = \frac{1}{3}\rho v_{rms}^2$

Equipartition of energy:  $K = \frac{1}{2}kT$  for each degree of freedom. Thus,  $K = \frac{f}{2}kT$  for molecule having  $f$  degrees of freedoms.

Internal energy of  $n$  moles of an ideal gas is  $U = \frac{f}{2}nRT$ .

### 4.3: Specific Heat

Specific heat:  $s = \frac{Q}{m\Delta T}$

Latent heat:  $L = Q/m$

Specific heat at constant volume:  $C_v = \left. \frac{\Delta Q}{n\Delta T} \right|_V$

Specific heat at constant pressure:  $C_p = \left. \frac{\Delta Q}{n\Delta T} \right|_p$

Relation between  $C_p$  and  $C_v$ :  $C_p - C_v = R$

Ratio of specific heats:  $\gamma = C_p/C_v$

Relation between  $U$  and  $C_v$ :  $\Delta U = nC_v\Delta T$

Specific heat of gas mixture:

$$C_v = \frac{n_1C_{v1} + n_2C_{v2}}{n_1 + n_2}, \quad \gamma = \frac{n_1C_{p1} + n_2C_{p2}}{n_1C_{v1} + n_2C_{v2}}$$

Molar internal energy of an ideal gas:  $U = \frac{f}{2}RT$ ,  
 $f = 3$  for monatomic and  $f = 5$  for diatomic gas.

### 4.4: Thermodynamic Processes

First law of thermodynamics:  $\Delta Q = \Delta U + \Delta W$

Work done by the gas:

$$\Delta W = p\Delta V, \quad W = \int_{V_1}^{V_2} p dV$$

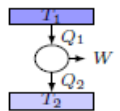
$$W_{\text{isothermal}} = nRT \ln \left( \frac{V_2}{V_1} \right)$$

$$W_{\text{isobaric}} = p(V_2 - V_1)$$

$$W_{\text{adiabatic}} = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$$

$$W_{\text{isochoric}} = 0$$

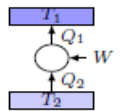
Efficiency of the heat engine:



$$\eta = \frac{\text{work done by the engine}}{\text{heat supplied to it}} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{\text{Carnot}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Coeff. of performance of refrigerator:



$$\text{COP} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

Entropy:  $\Delta S = \frac{\Delta Q}{T}$ ,  $S_f - S_i = \int_i^f \frac{\Delta Q}{T}$

$$\text{Const. } T : \Delta S = \frac{Q}{T}, \quad \text{Varying } T : \Delta S = ms \ln \frac{T_f}{T_i}$$

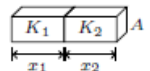
Adiabatic process:  $\Delta Q = 0$ ,  $pV^\gamma = \text{constant}$

### 4.5: Heat Transfer

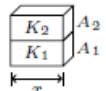
Conduction:  $\frac{\Delta Q}{\Delta t} = -KA \frac{\Delta T}{x}$

Thermal resistance:  $R = \frac{x}{KA}$

$$R_{\text{series}} = R_1 + R_2 = \frac{1}{A} \left( \frac{x_1}{K_1} + \frac{x_2}{K_2} \right)$$

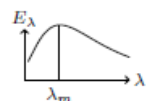


$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{x} (K_1A_1 + K_2A_2)$$



Kirchhoff's Law:  $\frac{\text{emissive power}}{\text{absorptive power}} = \frac{E_{\text{body}}}{a_{\text{body}}} = E_{\text{blackbody}}$

Wien's displacement law:  $\lambda_m T = b$



Stefan-Boltzmann law:  $\frac{\Delta Q}{\Delta t} = \sigma eAT^4$

Newton's law of cooling:  $\frac{dT}{dt} = -bA(T - T_0)$

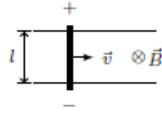
## 5.7: Electromagnetic Induction

Magnetic flux:  $\phi = \oint \vec{B} \cdot d\vec{S}$

Faraday's law:  $e = -\frac{d\phi}{dt}$

Lenz's Law: Induced current create a  $B$ -field that opposes the change in magnetic flux.

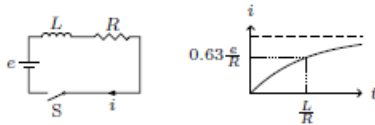
Motional emf:  $e = Blv$



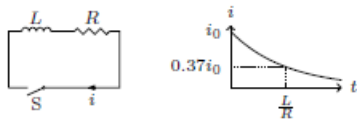
Self inductance:  $\phi = Li$ ,  $e = -L\frac{di}{dt}$

Self inductance of a solenoid:  $L = \mu_0 n^2 (\pi r^2 l)$

Growth of current in LR circuit:  $i = \frac{e}{R} [1 - e^{-t/LR}]$



Decay of current in LR circuit:  $i = i_0 e^{-t/LR}$



Time constant of LR circuit:  $\tau = L/R$

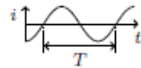
Energy stored in an inductor:  $U = \frac{1}{2} Li^2$

Energy density of  $B$  field:  $u = \frac{U}{V} = \frac{B^2}{2\mu_0}$

Mutual inductance:  $\phi = Mi$ ,  $e = -M\frac{di}{dt}$

EMF induced in a rotating coil:  $e = NAB\omega \sin \omega t$

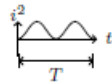
Alternating current:



$$i = i_0 \sin(\omega t + \phi), \quad T = 2\pi/\omega$$

Average current in AC:  $\bar{i} = \frac{1}{T} \int_0^T i dt = 0$

RMS current:  $i_{\text{rms}} = \left[ \frac{1}{T} \int_0^T i^2 dt \right]^{1/2} = \frac{i_0}{\sqrt{2}}$



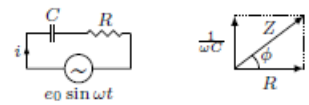
Energy:  $E = i_{\text{rms}}^2 RT$

Capacitive reactance:  $X_c = \frac{1}{\omega C}$

Inductive reactance:  $X_L = \omega L$

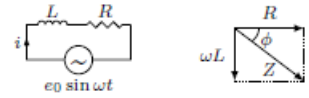
Impedance:  $Z = e_0/i_0$

RC circuit:



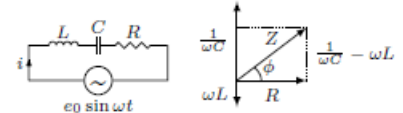
$$Z = \sqrt{R^2 + (1/\omega C)^2}, \quad \tan \phi = \frac{1}{\omega CR}$$

LR circuit:



$$Z = \sqrt{R^2 + \omega^2 L^2}, \quad \tan \phi = \frac{\omega L}{R}$$

LCR Circuit:

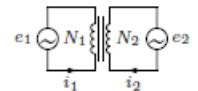


$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}, \quad \tan \phi = \frac{\frac{1}{\omega C} - \omega L}{R}$$

$$\nu_{\text{resonance}} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

Power factor:  $P = e_{\text{rms}} i_{\text{rms}} \cos \phi$

Transformer:  $\frac{N_1}{N_2} = \frac{e_1}{e_2}$ ,  $e_1 i_1 = e_2 i_2$



Speed of the EM waves in vacuum:  $c = 1/\sqrt{\mu_0 \epsilon_0}$